Consider the simply supported rectangular beam shown in Figure 1.15a. If a load were applied to the beam it would bend as shown exaggerated in Figure 1.15b. The deformation that takes place in bending causes the fibres above the neutral axis (NA) of the beam to shorten or compress and those below to stretch. In resisting this shortening and stretching the fibres of the beam are placed in compression and tension respectively. This induces compressive stresses above the NA and tensile stresses below. These are a maximum at the extreme fibres and zero at the NA, as indicated on the Figure 1.15b stress diagram.

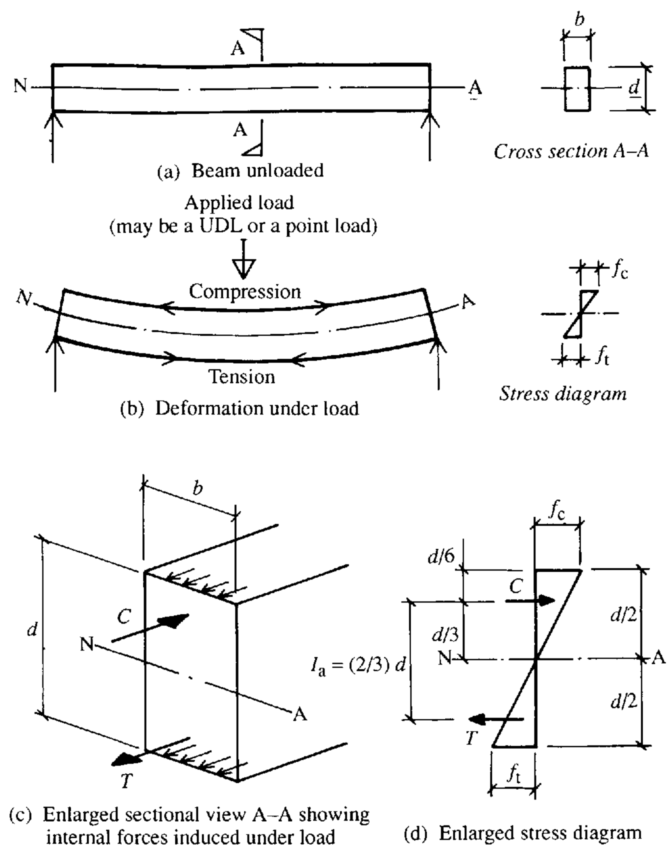

Figure 1.15 *Theory of bending related to a simply supported rectangular beam*

By reference to the enlarged beam cross-section Figure 1.15c and stress diagram Figure 1.15d, it can be seen that a couple is set up within the beam comprising a compressive force C and a tensile force T acting at the centres of gravity of the stress blocks with a lever arm of $(2/3)d$. The moment of resistance of the beam is the product of this couple:

$$\begin{aligned} \text{MR} &= \text{force } C \text{ (or } T) \times \text{lever arm} \\ &= \text{force } C \text{ (or } T) \times \tfrac{2}{3}d \end{aligned}$$

The forces C and T are equal to the average stress multiplied by the surface area upon which it acts. The average stress is either $f_c/2$ or $f_t/2$; the surface area is half the beam cross-section, that is $bd/2$. Therefore

$$\text{Force } C = \frac{f_c}{2} \times \frac{bd}{2} = f_c \frac{bd}{4}$$

$$\text{Force } T = \frac{f_t}{2} \times \frac{bd}{2} = f_t \frac{bd}{4}$$

hence

$$\text{MR} = f_c \frac{bd}{2} \times \frac{2}{3}d \quad \text{or} \quad f_t \frac{bd}{4} \times \frac{2}{3}d$$

$$= f_c \frac{bd^2}{6} \quad \text{or} \quad f_t \frac{bd^2}{6}$$

Since the values of f_c and f_t are the same, the symbol f may be adopted for the stress. So

$$\text{MR} = f \frac{bd^2}{6}$$

The term $bd^2/6$ is the section modulus Z, referred to earlier, for a rectangular beam of one material such as timber. Rectangular reinforced concrete beams are composite beams consisting of two materials and are therefore not within this category.

For rolled steel beams a value for the section or elastic modulus is obtained directly from steel section property tables. The values for the section modulus Z are given in length units^3 and those for the second moment of area in length units^4.

Let us now examine the use of the theory of bending for simple beam design.

Example 1.6

A timber beam spanning 4.5 m supports a UDL of 4 kN including its self-weight, as shown in Figure 1.16. Assuming the breadth of the beam to be 50 mm and the allowable stress in timber to be 7 N/mm^2, what depth of beam is required?

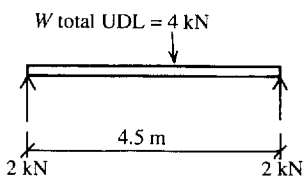

Figure 1.16 *Load diagram*

We have $b = 50$ mm and $f = 7$ N/mm^2; d is to be found. First,

$$\text{Internal MR} = \text{external BM maximum}$$

$$\frac{fbd^2}{6} = \frac{WL}{8}$$

now

$$\frac{fbd^2}{6} = \frac{7 \times 50 \times d^2}{6} \text{ N mm}$$

$$\frac{WL}{8} = \frac{4 \times 4.5}{8} \text{ kN m}$$